# Symbolic dynamics and topology. A few ideas on reconstruction of dynamics from finite time series

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In this talk we will present an idea on how to combine Takens' embedding theorem with tools from Topological Data Analysis in order to obtain a reconstruction of dynamics. The result of the considered process is a filtered transition matrix that can be subsequently analyzed with a variety of methods.

# Data, their shape and relations – why can we gain by understanding it?

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The concept of shape is often very fundamental. While common shapes can be very well understood by our brains, some of their basic mathematical descriptors have been proposed only recently. In this talk I will provide an intuitive introduction to topological data analysis – a field providing quantitative and qualitative descriptors of a shape of data of various dimensions. Initially I will focus on somewhat classical subjects like mapper and persistent homology, and later touch upon some highly effective algorithms to compute Euler characteristic profiles and mapper graphs representing relations between high dimensional datasets. A particular focus will be given to applications of topological data analysis in medical and physical sciences.

# Persistent homology and the analysis of heart rate variability

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Heart rate variability (hrv) is a physiological phenomenon of the variation in the length of the time interval between consecutive heartbeats. In many cases it could be an indicator of the development of pathological states. The classical approach to the analysis of hrv includes time domain methods and frequency domain methods. However, attempts are still being made to define new and more effective hrv assessment tools. Persistent homology is a novel data analysis tool developed in the recent decades that is rooted at algebraic topology. The Topological Data Analysis (TDA) approach focuses on examining the shape of the data in terms of connectedness and holes, and has recently proved to be very effective in various fields of research. In this paper we propose the use of persistent homology to the hrv analysis. We recall selected topological descriptors used in the literature and we introduce some new topological descriptors that reflect the specificity of hrv, and we discuss their relation to the standard hrv measures. In particular, we show that this novel approach provides a collection of indices that might be at least as useful as the classical parameters in differentiating between series of beat-to-beat intervals (RR-intervals) in healthy subjects and patients suffering from a stroke episode.

# Distributed algorithms for Euler Characteristic Curves (and Profiles)

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The Euler characteristic of a simplicial complex is the alternate sum of its Betti number, or equivalently the alternating sum of the number of its simplices of following dimensions. For a filtered complex the Euler Characteristic Curve is a function that assigns an Euler number to each filtration level. In the first part of this talk we will introduce the concept of ECC and discuss its stability with respect to the 1-Wasserstein distance. In the second part we will present new techniques to compute the ECC of filtered Vietoris-Rips or cubical complexes. By following a distributed approach, the contributions to the ECC can be computed locally without having to explicitly build up the whole complex. This allows us to significatively reduce both time and memory requirements, giving us the opportunity to tackle much larger datasets compared to, for instance, persistent homology. Such ideas and algorithms can be naturally extended to work in the multiparameter persistence setting, giving rise to the notion of Euler Characteristic Profiles.

# Multivector fields thoery for data analysis

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The recently developed multivector fields theory is an approach that provides means for a combinatorial representation of a vector field. It extends and is inspired by Forman's combinatorial vector fields that naturally arose from the discrete Morse theory. We incorporated multiple notions from the continuous theory of dynamical systems to make multivector fields a legible combinatorial counterpart of continuous vector fields. Among others, we adapted the concept of isolated invariant sets, Conley index theory, limit sets, and Morse decomposition. Except for the theoretical motivations, the theory is developed to be a new tool for analyzing empirical data with dynamical nature, e.g., a sampled vector field. To this end, we use persistence homology to study the structure and/or evolution of Morse decomposition or the Conley index.

In this talk, I will present the general idea of the multivector fields theory, some numerical experiments, and future ideas of its applications.

# Periodic and chaotic dynamics in a map-based neuron model

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Map-based neuron models are an important tool in modelling neural dynamics and sometimes can be considered as an alternative to usually computationally costlier models based on continuous or hybrid dynamical systems. However, due to their discrete nature, rigorous mathematical analysis might be challenging. We study a discrete model of neuronal dynamics introduced by Chialvo [Chaos, Solitons & Fractals 5, 1995, 461–479]. In particular, we show that its reduced one-dimensional version can be treated as an independent simple model of neural activity where the input and the fixed value of the recovery variable are parameters. This one-dimensional model still displays very rich and varied dynamics. Using the fact that the map whose iterates define voltage dynamics is S-unimodal, we describe in detail both the periodic behaviour and the occurrence of different notions of chaos, indicating corresponding regions in parameter space. Our study is also complemented by a bifurcation analysis of the mentioned dynamical model.

# Searching for most useful metrics in automatic comparison between radiotherapy treatment plans and high-definition 3D gel dosimetry images

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Radiotherapy works by killing tumor cells with radiation while sparing healthy cells, incl. organs at risk. Structures to be treated and to be avoided are distributed in a 3D space within a patient's body (a 3D, moving, deformable object). Tumor control and normal tissue complication probabilities are functions of several biologically-relevant variables, such as radiation dose (energy absorbed per unit mass), dose rate (dose per unit time), and linear energy transfer (average energy lost by ionizing particles per unit length of their track). Therefore, radiotherapy treatment planning involves a multidimensional problem. Verification of treatment delivery requires special dosimetry equipment with adequate sensitivity to those variables in a 3D space with adequate spatial resolution. Among many dosimetry systems, gel dosimetry is unique in its ability to generate accurate 3D data with millimeter spatial resolution. Gel dosimetry images are generated by transmission laser tomography (laser CT) or MRI scans of ,,treated" gel phantoms, i.e. tissue-equivalent patient-mimicking 3D objects where radiation triggers measurable material changes. Due to massive amount of data and high level or responsibility for treatment outcome, quantitative comparison of gel dosimetry data with an individual patient's treatment plan is normally a very time-consuming process, which is why this very promising approach has not yet been widely accepted in clinical practice. Any significant reduction of this time without sacrificing quality (or with improving it!) can improve outcomes by routinely detecting and correcting currently undetectable errors. This includes developing and establishing proper metrics in automatic comparison between treatment plans and 3D gel dosimetry images. The authors are inviting mathematicians to a scientifically exciting and clinically important collaborative effort in this direction.

# Topological analysis of breathing patterns

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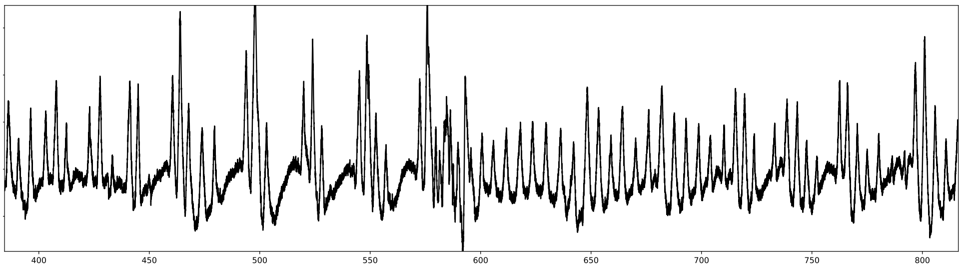
The input to our computational method is a recording of breathing of a patient over the time period of about 20 minutes. The recordings are obtained by means of a belt that essentially measures the circumference of the chest, which varies while the person breathes. This medical device will be shown during our visit at the Medical University of Gdańsk. In particular, the peak values correspond to the moment of maximum air intake. Here is a sample excerpt:




There are several methods that can be applied to the analysis of the time series obtained in our measurements. In our research, we compute persistent homology of short segments of the signal, and then we apply a machine learning technique in order to determine classes of different breathing patterns.

# Persistence-based descriptors for surface roughness and its relation to bio-interface characteristics

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One of the major complications in trauma and orthopaedic surgery is the implant-associated infection. Biofilm forming bacteria use the hydrophilic metal surfaces as adhesion area which provides them with a pathogenic advantage. Infections are caused by contamination of the metal surfaces being implanted and a subsequent colonisation of the bacteria which spread on the surface after building adhesion to it. In combination to several procedures and measures to prevent and reduce complications through implant-associated infections, surfaces specifically designed by roughness manipulation to minimise bacterial adhesion are an important research topic. Shot peening, in particular, is one of the more promising surface modification procedures. To help understand the connection of bacteria on-growth and the surface characterization better we need to have good descriptors of its surface morphology. Surface roughness descriptors as defined by the ISO may struggle however to properly describe surface roughness patterns. Descriptors based on the persistence diagrams of the surfaces on the other hand provide several advantages opposed to the standard roughness parameters.

# Topological-numerical analysis of a two-dimensional dynamical model of a neuron

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We conduct computer-assisted analysis of the two-dimensional model of a neuron introduced by Chialvo in 1995 (Chaos, Solitons & Fractals 5, 461–479). We apply the method for rigorous analysis of global dynamics based on a set-oriented topological approach, introduced by Arai et al. in 2009 (SIAM J. Appl. Dyn. Syst. 8, 757–789). Additionally, we introduce a new algorithm to analyze the return times inside a chain recurrent set, and together with the information on the size of the chain recurrent set, we develop a new method that allows one to determine ranges of parameters for which chaotic dynamics may appear.